Constraints on Topological Defect Formation in First-order Superconducting Phase Transitions

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In this work we address the impact of a cubic term addition to the Ginzburg-Landau mean-field potential, and study the consequences on the description of first order phase transitions in superconductors. Constraints are obtained from experiment and used to assess consequences on topological defect creation. No fundamental changes in either the Kibble-Zurek or Hindmarsh-Rajantie predictions are found.

I. INTRODUCTION

The following work is based on the research reported in Ref. [1]. It pursues the objective of empathizing the analogy between accessible condensed matter systems and the currently accepted framework for the evolution of the Universe, notably the symmetry-breaking phase transitions [2, 3] it has undergone after the Big-Bang. A key aspect to this comparison is the creation of topological defects, frustrations of the unbroken phase within the broken one, arising from the continuity of the order parameter values. These are generally categorized according to the homotopy group of the quocient of the unbroken symmetry groups to the broken one and which enables for comparison of different physical phenomena. These objects can appear as magnetic monopoles or point-like defects, cosmic strings, vortices or flux tubes, magnetic domain walls or textures. Besides its mere aesthetical value, this analogy can provide a powerful probe into the early stages of the evolution of the Universe, since direct, hands-on experimental tests are unattainable: the existence of more accessible systems that exhibit a formally similar behavior could provide crucial clues to many cosmologically relevant issues.

These "cosmology in the laboratory" experiments can be found in various systems, ranging from vortices in superfluid phase transitions of ^4He and ^3He (see e.g. Ref. [4, 5]), which exhibit common features with cosmic strings [6], to liquid crystals undergoing an isotropic-nematic phase transition [7, 8]. Polymer chains were shown to also possess analogous thermodynamic and transitional behavior [9]. However, most of these systems lack the existence of a quantity analogous to the magnetic field, which could be a key player in the early evolution of the Universe and formation of structure. For that reason, superconductors are a case of special interest. These comprise phase transitions involving a local gauge symmetry-breaking process, during which the photon acquires a "mass" and, therefore, a penetration length, giving rise to the Meissner effect: the expulsion of the magnetic field from a superconducting material, with formation of shielding "supercurrents" on its surface.

This symmetry breaking originates topological defects which are known as flux tubes or vortices, lines of non-null magnetic field trapped inside the superconductor. Experiments targeted at observing defect densities in high-T_c materials [10] were not in accordance with the density predictions of the Kibble-Zurek (K-Z) mechanism [2]. This, however, is to be expected, since the K-Z prediction should be accurate only for global gauge symmetry breaking, when the geodesic rule for phase angle summation is valid. A new defect generation mechanism, based on a local gauge treatment by Hindmarsh and Rajantie (H-R) [11], leads to an (additive) prediction. This, although well below the first Carmi-Polturak experimental sensitivity, is in reasonable agreement with the second.

As a starting point for this research, we note that the above experiments were both conducted in type-II materials, which exhibit a higher critical temperature and are therefore easier to manipulate, leading the current trend in experimental superconductivity. These materials display a second order phase transition, with no release of latent heat. On the other hand, Type-I materials are metastable, showing different responses to a magnetic field when in normal-superconductor or superconductor-normal phase transitions. In this work, we try to account for this more elaborate behavior and to estimate to which extent it affects the defect density predictions for type-I superconductors.

Type-I and type-II superconductors are commonly distinguished according to their Ginzburg-Landau (G-L) parameter $\kappa = \lambda/\xi$, the ratio between the magnetic field penetration length λ and the coherence length ξ of the order parameter. In the presence of a gauge field $\vec{A}$, these characteristic length scales are obtained from the free energy density

$$F(\Phi) = \frac{1}{2m_e} \left| i\hbar\vec{\nabla}\Phi - \frac{e}{c}\vec{A}\Phi \right|^2 + V(\Phi) + \frac{1}{2}\vec{\mu}\cdot(\vec{\nabla}\times\vec{A}) \ , \tag{1}$$

where $\vec{\mu}$ is the sample's magnetic moment, m_e is the electron mass and Φ is the order parameter. The G-L potential is usually written as [6]

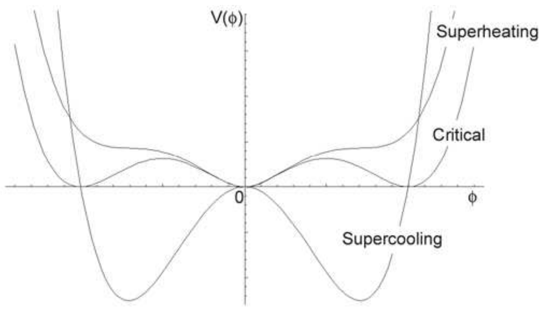

FIG. 1: Characteristic Potential curves.

$$V(\Phi) = \alpha\Phi^2 + \frac{\beta}{2}\Phi^4 \quad , \tag{2}$$

where α is assumed to be linear with temperature, $\alpha = \alpha'(t-1)$, $t \equiv T/T_c$, α' and β are constants, and T_c is the critical temperature. One obtains

$$\lambda = \sqrt{\frac{m_e c^2}{4\pi e^2}\frac{\beta}{|\alpha|}} \quad , \tag{3}$$

and

$$\xi = \hbar/\sqrt{2m_e|\alpha|} \quad . \tag{4}$$

The coherence length at zero temperature is $\xi_0 = \hbar/\sqrt{2m_e\alpha'}$, with $\kappa \sim \sqrt{\beta}$. The transition is second order if $\kappa > 1/\sqrt{2}$, and ξ_0 is typically less than $\sim 0.04\ \mu m$; for $\kappa < 1/\sqrt{2}$, the transition is first order and ξ_0 typically greater than $\sim 0.08\ \mu m$. It is commonly accepted that, if there is no applied magnetic field, one always has a second-order phase transition, for all values of κ. First order transitions arise from the external field term in Eq. (1) if the sample has a characteristic dimension $l > \lambda$. This degeneracy of the phase transition at $H = 0$ is, however, only verifiable to the current experimental sensitivity, and it can be argued that there is some yet undetected intrinsic metastability, regardless of the applied magnetic field.

Bearing in mind the analogy between condensed matter and cosmology, we now briefly look at phase transitions in high energy physics. In thermal field theory (TFT) a first order phase transition arises due to 1-loop radiative corrections to a potential similar to that of Eq. (2); a barrier between minima of the potential is created, as these corrections give rise to a cubic scalar field term

$$V(\Phi) = \alpha\Phi^2 - \gamma|\Phi|^3 + \frac{\beta}{2}\Phi^4 \quad , \tag{5}$$

where $\gamma(T) = (\sqrt{2}/4\pi)e^3T$ [12]. As before, β is assumed to be constant and $\alpha = \alpha'(t-1)$ to depend linearly with temperature.

In the normal-to-superconductor phase transition, a term similar to $-\gamma|\Phi|^3$ also arises if one takes into account gauge field fluctuations [13, 14], producing

$$\gamma = 8\mu_0\frac{e}{\hbar c}\sqrt{\pi\mu_0}T_c \quad . \tag{6}$$

This result enables a first order phase transition for all values of κ. Thermal fluctuations [14] and non local BCS effects [15] describe crossover behavior between first and second order transitions.

In the following, we adopt a potential of the form of Eq. (5) and constrain $\gamma(T)$ based on experimental data. The results are then compared with both TFT 1-loop radiative corrections and the the results of Ref. [13] (valid only at temperature close to T_c, that is, $t \to 1$). The introduction of $\gamma(T)$ in the potential (2) can produce changes in both the K-Z and H-R defect generating mechanisms. Also, a possible nucleation suppression due to the potential barrier can significantly reduce the number of observed defects. The obtained constraints on $\gamma(T)$ are used to access the impact on these claims.

II. TEMPERATURE SENSITIVITY BOUND

A superconductor undergoing a first-order phase transition crosses different supercritical fields, displaying a metastable behavior, as shown in the phase diagram of Figure 1. The superheating curve is given by the condition $\frac{dV}{d\Phi} = \frac{d^2V}{d\Phi^2} = 0$, for $\Phi \neq 0$, equivalent to $\alpha = 9\gamma^2/16\beta$. The supercooling curve is given by the condition $\frac{d^2V(\Phi)}{d\Phi^2} = 0$ for $\Phi = 0$, corresponding to $\alpha = 0$. The (unobservable) critical curve is given by $V(0) = V(\Phi_c)$ and $\frac{dV(\Phi_c)}{d\Phi} = 0$, where Φ_c is the non-vanishing minimum of the potential. This corresponds to $\alpha = \gamma^2/2\beta$.

Assuming $\alpha = \alpha'(t-1)$ and $\gamma(t) = \delta\ t$, we obtain for the superheating curve

$$\alpha'(t-1) = \frac{9}{16}\frac{\delta^2}{\beta}t^2 \quad , \tag{7}$$

and

$$t_{sh} = \frac{2}{1+\sqrt{1-\frac{9}{4}\frac{\delta^2}{\alpha'\beta}}} \sim 1 + \frac{9}{16}\frac{\delta^2}{\alpha'\beta} \quad . \tag{8}$$

Due to the presence of the cubic term in the potential of Eq. (5), the superheating curve shows a zero-field shift in temperature from T_c by $(9\delta^2/16\alpha'\beta)T_c$. This shift, if detected, would indicate an intrinsic metastability, in the sense that it does not depend on the existence of an applied field. Since such temperature shift has not yet been signaled, the current experimental temperature sensitivity being $\Delta t_{exp} \sim 10^{-3}$ [16], a bound on the slope of γ is

TABLE I: Critical properties of Sn and Al

Material	T_c (K)	$H_c(0)$ (G)	ξ_0 (μm)	λ (nm)
Sn	3.7	309	0.23	34
Al	1.2	105	1.6	16

$$\frac{9}{16}\frac{\delta^2}{\alpha'\beta} < \Delta t_{exp} \quad . \tag{9}$$

The supercooling transition still occurs at $t = 1$, the critical temperature; this is natural, since it is determined solely by $\alpha = 0$ (neglecting a smaller order correction to α, [13]).

III. SUPERHEATING PERTURBATION BOUND

To obtain further constraints, one looks more carefully at the mechanism relating the presence of an applied field with metastability. According to Ref. [17], this arises due to the contribution of the magnetic moment to the free energy (1), and hence it depends not only on the material (that is, on κ), but also on its shape and dimensions. The supercritical fields and the value of κ have commonly been obtained from experiments with microspheres of type-I materials. Table I indicates the critical properties of two of these, Sn and Al. To assess the influence of the cubic term of Eq. (5), we reproduce Ref. [17] calculations, including the cubic term in the potential. For a small sphere of radius a, the magnetic moment is given by [17]

$$\frac{\mu}{V} = -3\left[1 - \frac{3\lambda}{a\Phi_0}coth\frac{a\Phi_0}{\lambda} + \frac{3\lambda^2}{a^2\Phi_0^2}\right]\frac{H}{8\pi} \quad , \tag{10}$$

where $\Phi_0 \equiv \Phi/\Phi_\infty$ and $\Phi_\infty^2 \equiv m_e c^2/4\pi e^2\lambda^2$.

After a computation presented in [1], the reduced superheating field $h_{sh} \equiv H_{sh}/H_c$ is given by

$$h_{sh} = \left(1 + \frac{4}{\sqrt[4]{15}}\gamma_G\right)h_{sh}^0 \quad , \tag{11}$$

where $\gamma_G \equiv 3\gamma/2\sqrt{|\alpha|\beta}$ is defined to be dimensionless, while h_{sh}^0 is the unperturbed superheating field, corresponding to $\gamma = 0$.

Measurements [18, 19, 21, 22, 23] were obtained with colloidal dispersions, and the statistical error due to the size distributions of the microspheres do not allow for a direct fit of γ_G from $h_{sh}(t)$ data. The measurement reported in Ref. [20] used single microspheres, but nonlocality and impurities did lead to a large theoretical uncertainty in

TABLE II: Derived quantities and bounds for δ

Material	Sn	Al
α' (J)	1.15×10^{-25}	2.38×10^{-27}
β $(J.m^3)$	4.72×10^{-54}	2.16×10^{-56}
α' (eV^2)	3.61×10^{-1}	7.45×10^{-3}
β	9.45×10^{-4}	4.32×10^{-6}

bound	t_{sh} shift	$\delta(eV)$	t_{sh} shift	$\delta(eV)$
h_{sh}	0.25	1.23×10^{-2}	0.25	1.20×10^{-4}
ΔT_{exp}	10^{-3}	7.78×10^{-4}	10^{-3}	7.57×10^{-6}
Ref. [13]	5.19×10^{-9}	1.77×10^{-6}	2.32×10^{-6}	3.64×10^{-7}
TFT	2.92×10^{-9}	1.33×10^{-6}	3.24×10^{-6}	4.31×10^{-7}

$$h_{sh}^0(t) = \frac{1}{\sqrt{\kappa\sqrt{2}}}h_c^0(0)(1-t^2) \quad , \tag{12}$$

where $h_c^0(0) = H_c(0)/H_c(t)$, an approximation valid only close to T_c [24].

The calculation of the supercooling field implies the evaluation of a second derivative at the origin, and clearly the presence of a cubic term in the potential has no effect,

$$\frac{d^2}{d\Phi^2}(\gamma_G\Phi^3) = 6\gamma_G\Phi \to 0 \quad . \tag{13}$$

Since no shift of the superheating curve due to γ has been detected, we must have $\gamma_G \ll 1$. This constraint fails for relative temperatures in the range

$$1 - \frac{9}{4}\frac{\delta^2}{\alpha'\beta} < t < 1 \quad . \tag{14}$$

This interval is vanishingly small if

$$\frac{9}{4}\frac{\delta^2}{\alpha'\beta} \ll 1 \quad , \tag{15}$$

which is a weaker bound than the one of Eq. (9).

Al and Sn show maximum critical fields of order 10^2 G, and the shift between h_{sh} and h_{sh}^0 is smaller than 10^{-2} G. Since this is well below the sensitivity of measurements [18, 19, 20, 21, 22, 23], we consider only Eq. (9) and drop the bound of Eq. (15). Notice that, even if the "exclusion" interval" (14) is non-negligible, no "spikes" should appear in that region of the $H-T$ diagram, due to the smallness of the field values there.

Table II provides a comparison of the bounds on δ with the cubic term arising from 1-loop corrections in TFT and the prediction of Ref. [13]. This is achieved by computing the slope of $\gamma(t)$ from $\gamma(T) = (\sqrt{2}/4\pi)e^3T$,

obtaining $\delta = (\sqrt{2}/4\pi)e^3 T_c$. Notice that $\gamma(t)$ as a function of the reduced temperature is material dependent, although $\gamma(T)$ is not. The quantities α' and β are also included.

We notice that unit conversion is not direct, but achieved through a multiplicative factor m_e: since the dimension of the scalar field in G-L theory is $[\Phi^2] = L^{-3}$, its square representing a density, while in field theory $[\Phi] = L^{-1}$, the dimensionality of γ depends of the theory at hand. For comparison sake, we have chosen $[\gamma] = L^{-1}$. Hence, the definition of γ is changed with respect to the free energy potential of Eq. (5), through a convenient m_e factor; the electron mass determines the conversion as it is absent from the kinetic term of the Lagrangean density of field theory, $\partial_\mu \Phi \partial^\mu \Phi$, but present in the corresponding condensed matter free energy term, $(\hbar^2/2m_e)\nabla^2\Phi$; equivalently, one can loook at the coherence length: $\xi_{FT}^2 = 1/\alpha'$ *vs.* $\xi_{cm}^2 = \hbar^2/2m_e\alpha'$.

According to Ref. [13], in the absence of an applied magnetic field, momentum fluctuations of the gauge field have an expectation value derived from the equipartition theorem. Integrating over momentum space (with a cut-off Λ of the order of ξ_0^{-1}), one gets

$$\langle A^2 \rangle_\Phi = 4\frac{\mu_0}{\pi}\Lambda T_c - 8\mu_0 \frac{e}{\hbar c}\sqrt{\pi\mu_0}T_c|\Phi| \quad . \tag{16}$$

Hence, from this result arises, asides from an unimportant correction to the scalar field mass (or coherence length), a more relevant (negative) cubic term, $-8\mu_0(e/\hbar c)\sqrt{\pi\mu_0}T_c|\Phi|^3$. At zero field, this produces a shift in the superheating temperature of

$$\Delta_T = 7.25 \times 10^{-12} T_c^3 H_c(0)^2 \xi_0^6 \quad , \tag{17}$$

with $H_c(0)$ expressed in *Gauss* and ξ_0 in μm. For Sn, it requires a temperature sensitivity of $10^{-9}K$, well below current possibilities. Al, however, requires just a sensitivity of $10^{-6}K$, attainable if one employs state of the art relative temperature measurement techniques.

One can see that, for both materials, the slopes of γ predicted by TFT and Ref. [13] have similar magnitudes $\sim 10^{-7}eV$. This is a confirmation of the underlying analogous mechanisms: one can view the thermal averaging of the gauge field in condensed matter as equivalent to finite temperature vacuum polarization, given by renormalization of 1-loop Feynmann diagrams.

IV. TOPOLOGICAL DEFECT FORMATION

We now discuss possible implications of the presence of this cubic term in the mean-field potential: the K-Z mechanism predicts a topological defects (vortices) density of $n \simeq \xi_0^{-2}(\tau_0/\tau_q)^\nu$, where $\tau_0 = \pi\hbar/16k_BT_c$ is the characteristic time scale, given by the Gorkov equation, τ_q is the quench time, and ν is a critical exponent. One topological defect per ξ_0^2 area is assumed. Since the characteristic scales ξ_0 and λ are obtained by linearizing the G-L equations close to T_c, thus neglecting the γ-cubic and β-quartic terms contribution, we expect no significant changes to ξ_0 and, hence, to this prediction.

The defect density predicted by the H-R mechanism for a thin slab of width L_z is given by $n \simeq (e/2\pi)T^{1/2}L_z^{-1/2}\hat{\xi}^{-1}$, where $\hat{\xi} \sim 2\pi/\hat{k}$ is the domain size after the transition and is related to the highest wave number $\hat{k}$ to fall out of equilibrium. This is obtained from the adiabaticity relation

$$\left|\frac{d\omega(k)}{dt}\right| = \omega^2(k) \quad . \tag{18}$$

For an underdamped dispersion relation, one has $\omega(k) = \sqrt{k^2 + m_\gamma^2}$ and the photon mass is given by $m_\gamma^2 = 2e^2|\Phi|^2 = -2e^2\alpha/\beta$. This leads to a defect prediction $n \propto \tau_q^{-1/3}$. Recall that the penetration length λ expresses a non-null photon "mass" arising from the spontaneous symmetry breaking that occurs during the normal-to-superconductor phase transition. The presence of a cubic term in the potential will change the photon mass, as the true vacuum of the broken phase shifts to

$$\Phi = \frac{-3\gamma + \sqrt{-16\alpha\beta + 9\gamma^2}}{4\beta} \quad . \tag{19}$$

However, since $\gamma_G \equiv 3\gamma/2\sqrt{|\alpha|\beta} \ll 1$, the effect is, as in the K-Z case, too small to affect the H-R prediction in a significant way.

There is also a non-vanishing probability of the order parameter to quantum tunnel from the false vacuum towards the true one. The rate of transition per unit volume and time is given, in the thin wall approximation, [25, 26]

$$\frac{\Gamma}{V\Delta t} = T^4\left(\frac{S_3}{2\pi\, T}\right)^{3/2} e^{-S_3/T} \quad . \tag{20}$$

$$S_3(T) = \frac{2\pi}{81}\frac{1}{\beta^7\sqrt{\beta}}\frac{\gamma^9(T)}{\epsilon^2(T)} \tag{21}$$

is the Euclidean action, and $\epsilon(T)$ the "depth" of the true vacuum. For the thin wall approximation to be valid, one assumes that the barrier's height is much greater than this "depth" ϵ, which implies that γ is comparable to α and β, that is, $\gamma_G \sim 1$. Since, as shown above, the current temperature sensitivity of $10^{-3}K$ only allows for $\gamma_G < 10^{-2}$, the approximation breaks: the field should always tunnel through the potential barrier, that is, with a probability close to unity. Therefore, topological defect production is unsuppressed, and one does not need to be concerned that these may not have sufficient time to nucleate within the measurement's resolution time.

V. CONCLUSIONS

In this work we have examined a description of a type-I superconductive phase transition including a cubic term in the G-L mean-field potential, inspired both by analogy with TFT 1-loop corrections and gauge field thermal averaging [13]. Superheating field and temperature constraints impose bounds on this cubic term so that its effects are small compared to that of other parameters in the G-L potential. In particular, there is negligible impact on the defect density predictions of K-Z [2, 6] or H-R [11], with no suppression or slowing down of defect production due to nucleation suppression arising from the induced potential barrier between false and true vacua.

In the absence of any sizeable perturbations, the H-R prediction for topological defect density in type-I superconductors should be reduced by a factor of $10-100$. However, it has been suggested that defects nucleated in type-I materials survive significantly longer than in type-II [27]: for type-I superconductors, their lifetime is expected to be of order 10^{-4} seconds. This increases the possibility of measuring the created defects before they disappear, possibly compensating for an inferior net number.

Finally, we point out that the superheating temperature shift induced by a cubic term, derived either from Ref. [13] or from TFT 1-loop corrections, increases with decreasing κ ($\Delta t_{sh}(Sn) \sim 10^{-9}$; $\Delta t_{sh}(Al) \sim 10^{-6}$). Thus, possible searches for a cubic term should be conducted with extreme ($\kappa \ll 1$) type-I materials like α-tungsten, which displays $T_c = 15.4 \pm 0.5\ mK$, $H_c = 1.15 \pm 0.03\ G$. Also, much more precise bounds could be attained by using a DC SQUID to measure the shift in the supercritical field, since this device currently possesses a sensitivity of $10^{-5}\phi_0/\sqrt{Hz}$, or $10^{-6}G$ over a 10 μm grain diameter.

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